Enrollment No: ______________________   Exam Seat No:______________________

C.U.SHAH UNIVERSITY
Winter Examination-2015

Subject Name : Real Analysis

Subject Code : 4SC06RAC1   Branch :B.Sc.(Mathematics)

Semester : 6   Date :19/05/2016   Time : 02:30 To 05:30   Marks :70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1   Attempt the following questions: (14)

a) What is $\lim\inf a_n$? **(1)**

b) Is $\sum \frac{1}{n}$ divergent? **(1)**

c) Define: Convergent sequence. **(1)**

d) Following statement is true or false? Every Cauchy sequence is convergent. **(1)**

e) Define : $L(P, f)$. **(1)**

f) Following statement is true or false? Every convergent sequence is bounded. **(1)**

g) Write the name of any two test of positive term series. **(1)**

h) Find $\lim sup$ of {1,-1,1,-1…}. **(1)**

i) $\lim u_n = 2$, then $\sum u_n$ must be divergent. Following statement is true or false? **(1)**

j) What is primitive of function $f(x)$? **(1)**

k) Define: monotonic increasing sequence. **(1)**

l) What is sequence of partial sum of series? **(1)**

m) Define: Riemann Integrable function on $[a, b]$. **(1)**

n) Following statement is true or false? **(1)**

The series $\sum \frac{1}{n^e}$ is convergent where $e = 2.718$.

Attempt any four questions from Q-2 to Q-8

Q-2   Attempt all questions (14)

a) Find $\lim inf$ and $\lim sup$ for the following sequence. **(6)**

(I) $\{1 + (-1)^n\}$

(II) $\{n(-1)^n\}$

b) Using $\varepsilon - m$ definition show that $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$. **(5)**

(c) What is oscillating sequence .Explain the types of oscillating sequence with proper illustration. **(3)**

Q-3		**Attempt all questions**	**(14)**
	a)	State and Prove Cauchy's general principle of convergence for the sequence.	**(8)**
	b)	Prove that the sequence $\{s_n\}$, where $s_n = \left(1+\frac{1}{n}\right)^n$, is convergent and $lim\left(1+\frac{1}{n}\right)^n$ lies between 2 and 3.	**(6)**
Q-4		**Attempt all questions**	**(14)**
	a)	State and prove D'Alembert Ratio test for the convergence of series.	**(8)**
	b)	Test the convergence for the series. (1) $\frac{1}{2}+\frac{1}{2.2^2}+\frac{1}{3.2^3}+\frac{1}{4.2^4}+\ldots$ (2) $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\ldots$	**(6)**
Q-5		**Attempt all questions**	**(14)**
	a)	State and prove Leibnitz test for alternating series .	**(5)**
	b)	Define absolute convergence. Show that the series $\sum\frac{(-1)^{n+1}}{\sqrt{n}}$ is conditionally convergent .	**(5)**
	c)	State first mean value theorem for integral calculus.	**(4)**
Q-6		**Attempt all questions**	**(14)**
	a)	Show that the bounded function f is Integrable on $[a,b]$ if and only if for each $\varepsilon>0$ there exist $\delta>0$ and partition P with $\mu(P)<\delta$ and $U(P,f)-L(P,f)<\varepsilon$.	**(8)**
	b)	Show that 3x+1 is integrable on [1,2] and $\int_1^2(3x+1)dx=\frac{11}{2}$.	**(6)**
Q-7		**Attempt all questions**	**(14)**
	a)	If f_1 and f_2 are two bounded and integrable function on $[a,b]$ then show that $f=f_1+f_2$ is also bounded and integrable on $[a,b]$.	**(5)**
	b)	If a function f is monotonic on $[a,b]$, then show that it is integrable on $[a,b]$.	**(5)**
	c)	Find using Riemann sum $\int_{-1}^{1}\lvert x\rvert dx$.	**(4)**
Q-8		**Attempt all questions**	**(14)**
	a)	If f is bounded and integrable on $[a,b]$then $\lvert f\rvert$ is also bounded and integrable on $[a,b]$.	**(5)**
	b)	State and prove Fundamental theorem of calculus.	**(5)**
	c)	Show that: $\int_0^t \sin x\,dx = 1-\cos t$.	**(4)**